



# Influence of gravity modulation, viscous heating and concentration on flow past a vertical plate in slipflow regime with periodic temperature variations

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**ABSTRACT :** The influence of combined heat and mass transfer and gravity modulation on unsteady free convective, viscous incompressible flow past a porous vertical flat plate with periodic temperature in slip-flow regime retaining the terms involving viscous dissipation in the energy equation has been examined. Assuming variable suction at the plate and modulated gravity field of the form  $g = g_0 + g_1 \sin(\omega t)$ , the analytical expressions for flow characteristics are obtained. The effect of gravity modulation on the gravity profiles, temperature profiles, species concentration is discussed. Increase in gravity modulation parameter shows decrease in the value of skin-friction and has increasing effect on phase of skin-friction. Gravity modulation parameter has significant impact on Nusselt number. Even a small increase in gravity modulation parameter leads to significant increase in the value of Nusselt number and decrease in phase of heat transfer.

**Keywords :** Gravity modulation, Periodic temperature, Variable suction, Viscous dissipation.

## NOMENCLATURE

$A$	= suction parameter	$t$	= dimensionless time
$C$	= dimensionless species concentration in the fluid	$t^*$	= time
$C^*$	= species concentration in the fluid	$u$	= dimensionless velocity of fluid
$C_p$	= specific heat at constant pressure	$u^*$	= velocity of fluid
$C_\infty$	= species concentration in free stream	$V^*$	= suction velocity
$C_w^*$	= species concentration at the wall	$V_0^*$	= constant suction velocity of the fluid
$D$	= chemical molecular diffusivity	$\beta$	= coefficient of thermal expansion
$E_c$	= Eckert number	$\beta_0$	= coefficient of thermal expansion with concentration
$g$	= acceleration due to gravity	$\delta$	= phase angle for heat transfer
$g_0$	= constant gravity level	$\varepsilon$	= a constant ( $0 < \varepsilon \ll 1$ )
$g_1$	= the amplitude of oscillating component of acceleration due to gravity	$\phi$	= phase difference for skin friction
$G_c$	= modified Grashof number	$\kappa$	= thermal conductivity
$G_r$	= Grashof number	$\mu$	= viscosity
$L^*$	= constant	$\nu$	= kinematic viscosity
$N_u$	= dimensionless coefficient of heat transfer	$\rho$	= density of the fluid
$P$	= pressure	$\theta$	= dimensionless temperature
$P_r$	= prandtl number	$\omega$	= dimensionless temperature
$q$	= rate of heat transfer	$\omega^*$	= frequency of gravitational oscillation
$q_w^*$	= heat flux at the wall	$\tau$	= skin friction
$S_c$	= Schmidt number	$\tau_m$	= means skin friction
$T^*$	= temperature		
$T_\infty^*$	= temperature of fluid in free stream		
$T_w^*$	= temperature of wall		

## I. INTRODUCTION

Modern technology has enhance importance of free conversation flow land heat transfer problems due to its numerous and wide range of application, especially in the field of aeronautics, cooling devices and chemical engineering.

The phenomenon of free convection is the result of variation in density caused by change in temperature flows play an important role in chemical engineering, turbomachinery and aerospace technology. Recently there has been great interest in the study of the effect of complex body forces on fluid motion. Such forces can arise when a system with density gradients is subject to vibrations. The resulting buoyancy forces, which are produced by the interaction of density gradients with the acceleration field, have a complex spatiotemporal structure depending on both the nature of density gradients and the spatial and frequency distribution of the vibration-induced acceleration field. The effect of such forces on fluid motion known as gravity modulation or  $g$ -jitter has a profound effect on the homogeneous melt growth of semiconductor of metal crystals on earth-bound conditions. Theoretical efforts have been made by many researchers to explain the effects of the gravitational field on material processing inside a space shuttle environment. Jules [1] found that the international space station (ISS), is characterized by low mean accelerations which are  $O(10^{-6})g_c$  - the gravity on earth and fluctuations that are two or three order of magnitude above the mean. Recently many theoretical and experimental studies dealing with material processing or physics of fluids under the micro-gravity conditions aboard an orbiting spacecraft have shown that gravitational field can be resolved into a mean and fluctuating component. Space related technology has demanded a profound knowledge of forces involving vibrations that occur due to interaction of several phenomena. On spacecraft fluctuating accelerations can originate from a variety of sources such as crew activities, vibration from on-board equipment and structural oscillations of the spacecraft. The fluctuating acceleration act on density gradients in the fluid caused by heat and mass transfer between the fluid and boundaries to produce convective motions. These motions may lead to increase in heat transfer significantly beyond that of pure conduction.

Kamotami [2] performed a linearized stability analysis to show that gravity modulation can significantly affect the stability limits of the system. They observed that modulation normal to the direction of temperature gradient to be most critical. Wadih & Roux [3] in their study reported that vibrations can either substantially enhance or retard heat transfer and thus affect the convections. Biringen and Peltier [4] investigated the effects of three-dimensionality as well as non-linearity of the governing equations for both sinusoidal and random modulation and found that sinusoidal modulations are more stabilizing and random modulation. Duval and Jacqmin [5] considered the  $g$ -jitter convection of two diffusing miscible liquids of differing densities under an oscillating vertical gravitational field with zero mean. Ramaswamy [6] studies the effects of periodic source ( $g$ -jitter) on fluids system and heat transfer mechanism of convection flow by imposing sinusoidal gravity modulation field in two-dimensional Rayleigh-Benard problem. Cleaver [7] used Galerkin method to study the two-dimensional modes of oscillatory convection in

a gravitationally modulated fluid layer with rigid, isothermal boundaries heated either from below or from above in the case of both linear and nonlinear phenomena. Farooq & Homsy [8] analyzed the effect of gravity modulation was observed for sufficiently large modulation amplitude. Takako [9] using finite difference method investigated natural convection in a cylindrical cavity with sinusoidal gravity modulation. Chen & Chen [10] investigated the stability of convection in a differentially heated vertical slot under a modified gravity field  $g = g_0 + g_1 \cos(\omega t)$ . It is shown there that gravity modulation can stabilize or destabilize the flow.

Rees and Pop [11] studied the effect of a small but fluctuating gravitational field, characteristic of  $g$ -jitter, on the flow near the forward stagnation of a two dimensional symmetric body resulting from a step change in its surface temperature. It has been found that the parameters like Prandtl number, the forcing amplitude, and the forcing frequency affect considerably the shear stress and the rate of heat transfer. Shu [12] presented a finite element. Study of double-diffusive convection driven by  $g$ -jitter in a microgravity environment. Numerical study indicates that with an increase in  $g$ -jitter force (or amplitude), the nonlinear convective effects become much more obvious, which in turn drastically change the concentration fields. Christov and Homsy [13] studied in detail the non-linear dynamics of two-dimension convection in vertically stratified slot with and without gravity modulation. Shridan [14] studies the effect of  $g$ -jitter induced combined heat and mass transfer by mixed convection flow in microgravity for simple system consisting of two heated vertical parallel infinite flat plates held at constant but different temperatures and concentrations. The governing equations are solved analytically for the induced velocity, temperature and concentration distributions. Sharidan [15] studied the effect of periodical gravity modulation, or  $g$ -jitter induced mixed convection, on the flow and heat transfer characteristics associated with a stretching vertical surface in a viscous and incompressible fluid. The velocity and temperature of the sheet are assumed to vary linearly with the distance along the sheet. Siddavaram and Homsy [16] analyzed the effects of harmonic gravity modulation on fluid mixing. They investigated the physical mechanism by which  $g$ -jitter affects the mixing characteristics of two miscible fluids which initially meet at a sharp vertical interface. In a subsequent study Siddavaram & Homsy [17] analyzed the effects of stochastic gravity modulations the mixing characteristics of two inter-diffusion miscible Boussinesq fluids initially separated by a thin diffusion layer. Here the gravity modulation has a Gaussian probability distribution and is characterized by an exponentially damped cosine auto correlation function. Dyko and Vafai [18] investigated the effects of gravity modulation on convection in the annulus between two horizontal coaxial cylinders in microgravity environment to study the unsteady flow structures in a large-gap annulus; the three-dimensional transient equations of fluid motion and heat transfer were solved. Their work described convection in cylindrical annulus

under microgravity and provided practical information on the influence of gravity fluctuations on heat transfer in space environment Sharidan [19] presented a numerical solution for the effect of a small fluctuating gravitational field characteristic of  $g$ -jitter on free convection boundary layer flow near stagnation point region of a three-dimensional body. It has been shown that  $g$ -jitter affects considerably the flow characteristics, namely the skin friction and the rate of heat transfer.

In the present work the unsteady free convective flow of a viscous incompressible fluid past an infinite vertical flat porous plate in a slip-flow regime, with variable suction and gravity modulation are considered. The influence of combined heat and mass transfer and gravity modulation on unsteady free convective, viscous incompressible flow past a porous vertical flat plate with period temperature in slip-flow regime remaining the terms involving viscous dissipation in the energy equation has been examined. Assuming variable suction at the plate and modulated gravity field of the form  $g = g_0 + g_1 \sin(\omega t)$ , the analytical expressions for flow characteristics are obtained. Perturbation method has been used to obtain the solution. It has been noted that the gravity modulation makes a significant change in skin-friction and heat transfer.

## II. FORMULATION OF THE PROBLEM

An unsteady free convective flow of a viscous incompressible fluid past an infinite vertical flat porous plate in a slip-flow regime, with variable suction in the form  $V^* = -V_0^*(1 + \varepsilon A e^{i\omega t^*})$  and gravity acceleration in the form  $g = g_0 + g_1 \sin(\omega t)$  are considered. It is further assumed that the temperature of the plate oscillates in time about a non-zero constant mean. It is convenient to introduce a co-ordinate system with wall lying vertically in  $x^*$ - $y^*$  plane. The  $x^*$ -axis is taken normal to it. Since the plate is considered infinite in the  $x^*$ -direction, all physical quantities will be independent of  $x^*$ . Under these assumption, the physical variables are function of  $y^*$  and  $t^*$  only. Then considering viscous dissipation, concentration variation and assuming variation of density in the body force terms, the problem is governed by the following set of equation:

$$\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*} = g \beta (T^* - T_\infty^*) + g \beta_0 (C^* - C_\infty^*) + \nu \frac{\partial^2 u^*}{\partial y^{*2}} \quad \dots(1)$$

$$\frac{\partial T^*}{\partial t^*} + V^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial y^{*2}} \right) + \frac{a\nu}{C_p} \left( \frac{\partial u^*}{\partial y^*} \right)^2 \quad \dots(2)$$

$$\frac{\partial C^*}{\partial t^*} + V^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \quad \dots(3)$$

The variable suction on the vertical plate is imposed in the form

$$V^* = -V_0^*(1 + \varepsilon A e^{i\omega t^*}) \quad \dots(4)$$

The time-dependent gravitational acceleration is assumed in the form  $g = g_0 + g_1 \sin(\omega t^*)$ , where  $g_0$  is the constant gravity level in the space,  $g_1$  is the amplitude of the

oscillating component of acceleration. The gravitational acceleration is rewritten in the form

$$g = g_0 - i g_1 e^{i\omega t^*} \quad \dots(5)$$

It is assumed that the real part alone is physically relevant and the  $(*)$  stands for dimensional quantities.

The boundary conditions of the problem are :

$$u^* = L^* \frac{\partial y^*}{\partial y^*}, \quad T^* = T_w^* + \varepsilon (T_w^* - T_\infty^*) e^{i\omega t^*}$$

$$C^* = C_w^* + \varepsilon (C_w^* - C_\infty^*) e^{i\omega t^*} \quad \text{at } y^* = 0 \quad \dots(6)$$

$$u^* \rightarrow 0, \quad T^* \rightarrow T_\infty^*, \quad C^* \rightarrow C_\infty^* \quad \text{as } y^* \rightarrow \infty. \quad \dots(7)$$

All the physical variables are defined in Nomenclature. The subscript  $(\infty)$  and  $(w)$  denotes the free stream condition and conditions on the surface of plate respectively.

The following non-dimensional quantities are introduced:

$$y = \frac{y^* V_0^*}{\nu}, \quad t = \frac{t^* V_0^{*2}}{4\nu}, \quad \omega = \frac{4\nu\omega^*}{V_0^{*2}},$$

$$u = \frac{u^*}{V_0^*}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}$$

$$Gr \text{ (Grashof Number)} = g_0 \beta \nu \frac{T_w^* - T_\infty^*}{V_0^{*3}}$$

$$Gc \text{ (Modified Grashof Number)} = g_0 \beta_0 \nu \frac{C_w^* - C_\infty^*}{V_0^{*3}}$$

$$Pr \text{ (prandtl Number)} = \frac{\mu C_p}{k} = \frac{\nu}{(k/\rho C_p)}$$

$$Ec \text{ (Eckert number)} = \frac{V_0^{*2}}{C_p (T_w^* - T_\infty^*)}$$

$$Sc \text{ (Schmidt number)} = \frac{\nu}{D}$$

$$h \text{ (Rarefaction parameter)} = \frac{V_0^* L^*}{\nu}$$

Using (4) to (7) and the non-dimensional quantities in (1) to (3) yield

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = (1 - i\varepsilon \alpha e^{i\omega t}) (\theta Gr + CGc) + \frac{\partial^2 u}{\partial y^2} \quad \dots(8)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + a Ec \left( \frac{\partial u}{\partial y} \right)^2 \quad \dots(9)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad \dots(10)$$

where  $g_1 = \varepsilon \alpha g_0$ . The constant  $a$  has been arbitrarily introduced to compare the corresponding available results with the case of forced convection by taking it as zero otherwise it will be taken as 1.

The boundary condition (6) and (7) in the dimensionless form are

$$u = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{i\omega t},$$

$$C = 1 + \varepsilon e^{i\omega t}, \quad \text{at } y = 0 \quad \dots(11)$$

$$u \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad \dots(12)$$

### III. SOLUTION OF THE PROBLEM

Assuming the small amplitude oscillation ( $0 < \varepsilon \ll 1$ ), the velocity  $u$ , temperature  $\theta$  and concentration  $C$  near the plate are represented as

$$u(y,t) = y_0(y) + \varepsilon e^{i\omega t} u_1(y) \quad \dots(13)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) \quad \dots(14)$$

$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) \quad \dots(15)$$

Substituting (13) to (15) in (8) to (10) and comparing the coefficients of identical power of  $\varepsilon$ , we obtain

$$u_0'' + u_1' = -\theta_0 Gr - C_0 Gc \quad \dots(16)$$

$$u_1'' + u_1' - \frac{i\omega}{4} u_1 = -\theta Gr - C_1 Gc - Au_0' + i\alpha(\theta_0 Gr + C_0 Gc) \quad \dots(17)$$

$$\theta_0'' + \text{Pr}\theta_0' = -a Ec \text{Pr} (u_0')^2 \quad \dots(18)$$

$$\theta_1'' + \theta_1' \text{Pr} - \frac{i\omega}{4} \theta_1 \text{Pr} = -A \theta_0' \text{Pr} - 2au_0' u_1' Ec \text{Pr} \quad \dots(19)$$

$$C_0'' + Sc C_0' = 0 \quad \dots(20)$$

$$C_1'' + C_1' Sc - \frac{i\omega}{4} C_1 Sc = -AC_0' Sc \quad \dots(21)$$

where prime denotes derivative with respect to  $y$ .

The corresponding boundary condition (11) and (12) reduce to the form

$$u_0 = h \frac{\partial u_0}{\partial y}, \quad u_1 = h \frac{\partial u_1}{\partial y}, \quad \theta_0 = \theta_1 = 1, \quad C_0 = C_1 = 1, \quad \text{at } y = 0 \quad \dots(22)$$

$$u_0 \rightarrow 0, \quad u_1 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \quad C_0 \rightarrow 0, \quad C_1 \rightarrow 0, \quad \text{as } y \rightarrow \infty \quad \dots(23)$$

The equation (13) to (21) are still coupled for the variables  $u_0, u_1, \theta_0, \theta_1, C_0$  and  $C_1$ . These equation have been solved for  $Ec \ll 1$ , using perturbation technique in the following form

$$(u_0, \theta_0, C_0, \theta_1, C_1) = (u_{01}, \theta_{01}, C_{01}, u_{11}, \theta_{11}, C_{11})$$

$$+ Ec(u_{02}, \theta_{02}, C_{02}, u_{12}, \theta_{12}, C_{12}) \quad \dots(24)$$

Using (24) in (16) to (21) and comparing the coefficient of identical powers of  $Ec$ , the resulting equation are solved for evaluating  $u_{01}, \theta_{01}, C_{01}, u_{11}, \theta_{11}, C_{11}, u_{02}, \theta_{02}, C_{02}, u_{12}, \theta_{12}$  and  $C_{12}$ .

The velocity, temperature and the concentration up to first order terms in  $Ec$  are expressed in terms of the steady and fluctuating parts as

$$u(y, t) = u_{01}(y) + u_{02}(y) Ec + \varepsilon [u_{11}(y) + u_{12}(y) Ec] e^{i\omega t} \quad \dots(25)$$

$$\theta(y, t) = \theta_{01}(y) + \theta_{02}(y) Ec + \varepsilon [\theta_{11}(y) + \theta_{12}(y) Ec] e^{i\omega t} \quad \dots(26)$$

$$C(y,t) = C_{01}(y) + C_{02}(y) Ec + \varepsilon [C_{11}(y)$$

$$+ C_{12}(y) Ec] e^{i\omega t} \quad \dots(27)$$

### IV. RESULTS AND DISCUSSION

The effect of variable suction, gravity modulation and oscillating plate temperature on the velocity and temperature in slip flow regime is presented in the subsequent subsections. For physical interpretation of the problem, the numerical values of the velocity profiles and temperature profile have been computed at time  $t = \pi/4\omega$  for certain fixed values of physical parameter Prandtl number  $\text{Pr} = 0.71$ , which represents air at  $20^\circ\text{C}$  at 1 atmosphere pressure. The value of the Schmidt number  $Sc = 0.94$  is taken to represent the presence of species Carbon dioxide. In gravity modulation  $g = g_0 + g_1 \sin(\omega t)$ ,  $g_1/g_0 = \varepsilon \alpha = 10$  is considered. The value of Eckert number  $Ec$  is taken 0.001 and  $\varepsilon = 0.001$ . The values of Grashof number  $Gr$ . Modified Grashof number,  $Gc$  and rarefaction parameter  $h$  are selected arbitrarily. For cooling of the plate by free convection current,  $Gr > 0$  is taken. During the investigation, it has been found that fluctuations are confined near the plate.

#### Velocity profiles

The velocity profiles for carbon dioxide in air are shown in Fig. 1. It is observed from the figure that velocity component increases with increasing the rarefaction parameter  $h$  and decreases with increasing frequency  $\omega$  as well with increasing suction parameter  $A$ . Furthermore the increase  $Gr$  or  $Gc$  leads to increase in the velocity component, while reverse effect is observed for increase in value of gravity modulation parameter  $\varepsilon \alpha$ . Increase in  $Gc$ . The has more impact on the increase of velocity component as compared to increase in  $Gr$ . The velocity component shows more variation in the vicinity of the plate and then decreases exponentially far away from the plate. The particular case of velocity profiles of carbon dioxide in air for  $Gr = 2, Gc = 5, h = 0.4, A = 5, \omega = 10, \omega t = 0, \varepsilon = 0.2, a = 1$ , and  $\alpha = 0$  in the present study agrees with Sharma [20]. Also the results have been obtained numerically using MATLAB software. The numerical value have been compared with the values obtained by regular perturbation method for velocity profile as given in Table 1. The values show good agreement between analytical results and numerical results for the values of parameters used in computation.

#### Temperature Profiles

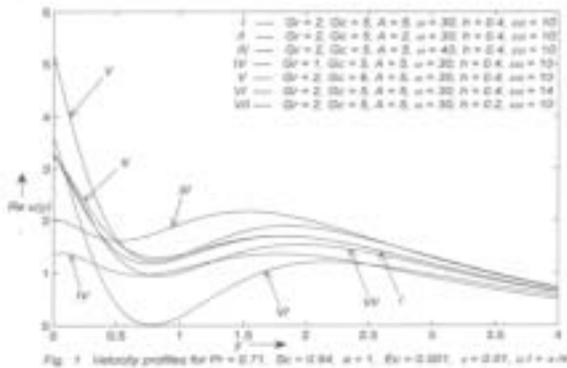
The temperature profiles are presented in Table 2. The temperature component decreases exponentially far away from the plate. It is observed that is slight increase in temperature component with increase in the rarefaction parameter  $h$  and with increase in frequency  $\omega$ . Furthermore the increase in  $Gr$  or  $Gc$  leads to increase in the temperature component, while reverse effect is observed for increase in value of gravity

**Table 1: Comparison of Analytical and Numerical values of Velocity profiles for  $Pr = 0.71, Gr = 2, Sc = 0.94, Gc = 5, \epsilon\alpha = 10, A = 5, a = 1, h = 0.4, Ec = 0.001, \epsilon = 0.01, \omega = 30, \omega t = \pi/4$ .**

u	y	0	0.5	1.0	1.5	2.0	2.5	3.0
Analytical		3.2888	1.4116	1.2747	1.6277	1.6749	1.4611	1.1665
Numerical		3.2874	1.4108	1.2741	1.6274	1.6745	1.4610	1.1664

**Table 2: The temperature component  $\theta$  at  $\omega t = \pi/4$  for  $Sc = 0.94, Ec = 0.001, A = 0, \epsilon = 0.01$  and  $Pr = 0.71$ .**

y	Gr=2 Gc=5 h=0.4 ω=30 εα=10	Gr=2 Gc=5 h=0.4 ω=30 εα=10	Gr=1 Gc=5 h=0.4 ω=30 εα=10	Gr=2 Gc=5 h=0.4 ω=30 εα=10	Gr=2 Gc=5 h=0.4 ω=30 εα=10	Gr=2 Gc=5 h=0.2 ω=30 εα=10
0	1.0071	1.0071	1.0071	1.0071	1.0071	1.0071
0.5	0.7031	0.7037	0.7031	0.7034	0.7027	0.7028
1.0	0.4921	0.4926	0.4917	0.4926	0.4918	0.4920
1.5	0.3453	0.3460	0.3447	0.3459	0.3450	0.3452
2.0	0.2430	0.2435	0.2423	0.2436	0.2428	0.2428
2.5	0.1711	0.1714	0.1704	0.1717	0.1710	0.1709
3.0	0.1204	0.1206	0.1198	0.1210	0.1204	0.1202
3.5	0.0847	0.0848	0.0841	0.0851	0.0847	0.0845
4.0	0.0595	0.0596	0.0591	0.0598	0.0595	0.0594



modulation parameter  $\epsilon \alpha$ . The gravity modulation has very little impact on the temperature profile as compared to velocity profile. For large value of gravity modulation parameter  $\epsilon \alpha$ , even the small frequency  $\omega$  has significant impact on the temperature profile. A particular case of temperature profiles for carbon dioxide in air with  $Gr = 2, Gc = 5, h = 0.4, A = 5, \omega = 10, \omega t = 0, \epsilon = 0.2, a = 1$  and  $\alpha = 0$  in the present study agrees with Sharma [20].

**Concentration Profiles**

Concentration profiles decrease exponentially far away from the plate. Change in the value of the suction parameter or frequency  $\omega$  and of gravity modulation parameter  $\epsilon \alpha$  has no impact on the concentration profiles.

**Skin-friction**

The skin-friction on the plate is defined as

$$\tau_x^* = \mu \frac{\partial u^*}{\partial y^*} \tag{28}$$

where  $\mu$  is viscosity.

The skin-friction  $\tau$  in the non-dimensional form on the plate  $y = 0$  is given by

$$\tau = \frac{\tau_x^*}{\rho V_0^{*2}} = \tau_m + \epsilon |M| \cos(\omega t + \phi) \tag{29}$$

where mean Skin friction ( $\tau_m$ ) is given by

$$\tau_m = -B_7 + Pr B_5 + Sc B_6 - Ec (B_{21} + Pr B_{22} + 2 B_{23} + 2 Pr B_{24} + 2 Sc B_{25} + (1+Pr) B_{26} + Pr+ Sc) B_{27} (1 + Sc) B_{28} \tag{30}$$

and unsteady component of skin friction  $M = M_r + iM_i$  is given by

$$M = -m_3 B_{15} - B_{16} + m_1 B_{17} + m_2 B_{18} + Pr B_{19} + Sc B_{20} - Ec [m_3 B_{46} - m_1 B_{47} + 2 B_{48} + Pr B_{49} + 2 Pr B_{50} + B_{51}$$

$$\begin{aligned}
 &+ (1 + Pr) B_{52} + 2 Sc B_{53} \\
 &+ (1 + Sc) B_{54} + (Pr + Sc) B_{55} + (1 + m_1) B_{56} \\
 &+ (1 + m_2) B_{57} + (1 + m_3) B_{58} \\
 &+ (m_1 + Pr) B_{59} + (m_2 + Pr) B_{60} + (m_3 + Pr) B_{61} \\
 &+ (m_1 + Sc) B_{62} \\
 &+ (m_2 + Sc) B_{63} + (m_3 + Sc) B_{64} \quad \dots(31)
 \end{aligned}$$

where phase angle  $\phi$  for skin-friction is evaluated from  $M_r$  and  $M_i$  which represent real and imaginary parts of  $M$  respectively. The constants occurring in the above equation have not been recorded here for the sake of brevity.

Fig. 2 shows  $\tau$ , the skin friction at  $\omega t = \pi/4$ . Skin-friction decreases with increasing rarefaction parameter  $h$  near the plate but it increases with increasing rarefaction parameter away from the plate. An increase in  $Gr$  or  $Gc$  leads to decrease in  $\tau$ , and the same effect is observed for increase in gravity

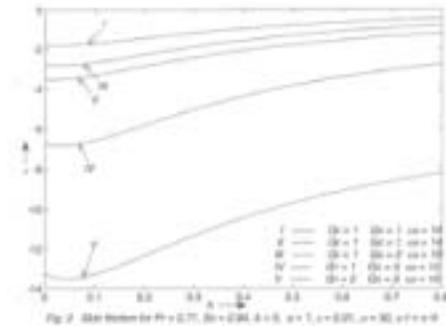


Fig. 2 Skin friction for  $Pr = 0.71, Sc = 0.64, \alpha = 1, \omega = 0.01, \omega = 0.01$

modulation parameter  $\epsilon \alpha$ . The phase of skin-friction for carbon dioxide in air is depicted by Fig. 3. It has been noted that the phase of skin-friction decreases with increasing suction parameter  $A$  as well as with increase in rarefaction parameters  $h$ . The phase of skin-friction decreases with increasing  $Gr$  or  $Gc$  for the same value of rarefaction parameter  $h$ . The phase of skin-friction increases with increase in value of gravity modulation parameter  $\epsilon \alpha$ .

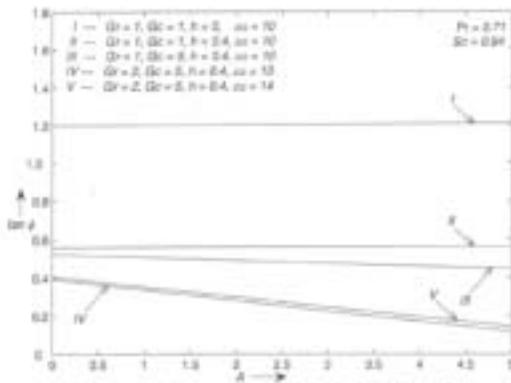


Fig. 3 Phase of transient component of skin friction for  $Gr = 0.001, \alpha = 0.01, \omega = 0.01$

### Coefficient of heat Transfer

Dimensionless coefficient of heat transfer is given by

$$Nu = \frac{q_w^* v}{kV_0^* (T_w^* - T_\infty^*)} = - \left. \frac{\partial \theta}{\partial y} \right|_{y=0} \quad \dots(32)$$

$$\text{where } q_w^* = -k \left. \frac{\partial T^*}{\partial y^*} \right|_{y=0}$$

In terms of amplitude and phase the rate of heat transfer can be written as

$$\begin{aligned}
 Nu = &Pr - Ec[-Pr B_8 + 2 B_9 + 2 Pr B_{10} + 2 Sc B_{11} + (1 + Pr) B_{12} \\
 &+ Pr + Sc) B_{13} + (1 + Sc) B_{14}] + \epsilon |H| \cos(\omega t + \delta) \quad \dots(33)
 \end{aligned}$$

where

$$\begin{aligned}
 H = &m_1 B_1 + Pr B_2 + B_{21} Ec[-m_1 B_{29} + 2 B_{30} + Pr B_{31} + 2 Pr \\
 &B_{31} \\
 &+ (1 + Pr) B_{33} + 2 Sc B_{34} \\
 &+ (1 + Sc) B_{35} + (Sc + Pr) B_{36} + (1 + m_1) B_{37} + (1 + m_2) B_{38} \\
 &+ (1 + m_3) B_{39} + (m_1 + Pr) B_{40} + (m_2 + Pr) B_{41} + (m_3 + Pr) \\
 &B_{42} + (m_1 + Sc) B_{43} + (m_2 + Sc) B_{44} + (m_3 + Sc) B_{45} \quad \dots(34)
 \end{aligned}$$

where phase angle  $\delta$  for heat transfer is given by  $H_r$  and  $H_i$  which represent real and imaginary parts of  $H$  respectively. The constants occurring in the above equations have not been recorded here for the sake of brevity.

Fig. 4 shows  $Nu$ . The Nusselt number at  $\omega t = \pi/4$ . With increasing suction parameter, there is monotonic increase in  $Nu$ . Nusselt number increases with increasing rarefaction parameter  $h$ . An increase in  $Gc$  leads to decrease and an increase in  $Gr$  lead to an increase in Nusselt number. Gravity modulation parameter has significant impact on Nusselt number. Even a small increase in gravity modulation parameter  $\epsilon \alpha$  leads to significant increase in the value of  $Nu$ . The numerical values of the phase difference  $\tan \delta$  are depicted in Fig. 5. Phase difference  $\tan \delta$  decreases with increasing suction parameter  $A$ , while increase in rarefaction parameter  $h$  shown decrease in phase difference near the plate but it shows reverse effect away from plate. With increasing  $Gr$  or  $Gc$ , For the same value of rarefaction parameter  $h$ ,  $\tan \delta$  decreases and same effect is observed for increasing value gravity modulation parameter  $\epsilon \alpha$ .

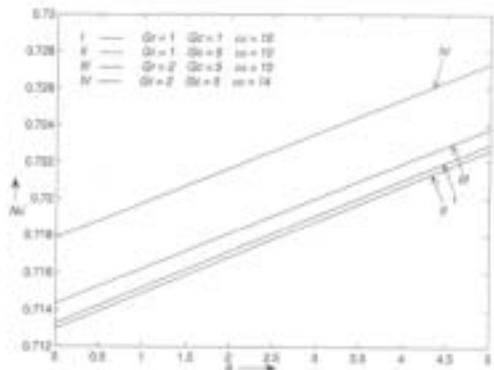


Fig. 4. Nusselt number for  $Pr = 0.71$ ,  $Sc = 0.24$ ,  $\alpha = 1$ ,  $\beta = 0.24$ ,  $Gr = 0.001$ ,  $\omega = 30$ ,  $\omega = 10$

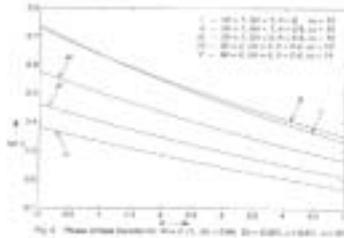


Fig. 5. Phase difference of Nusselt number for  $Pr = 0.71$ ,  $Sc = 0.24$ ,  $\alpha = 1$ ,  $\beta = 0.24$ ,  $Gr = 0.001$ ,  $\omega = 30$ ,  $\omega = 10$

## V. CONCLUSIONS

1. It is observed from the figure that velocity component increases with increasing the rarefaction parameter  $h$  and decreases with increasing frequency  $\omega$  as well as with increasing suction parameter  $A$ . Furthermore the increase in  $Gr$  or  $Gc$  leads to increase in the velocity component, while reverse effect is observed for increase in value of gravity modulation parameter  $\epsilon \alpha$ . Increase in  $Gc$  has more impact on the increase of velocity component as compared to increase in  $Gr$ . The velocity component shows more variations the vicinity of the plate and then decreases exponentially far away from the plate.

2. The gravity modulation has very little impact on the temperature profile as compared to velocity profile. For large value of gravity modulation parameter  $\epsilon \alpha$ , even the small frequency  $\omega$  has significant impact on the temperature profile.

3. An increase in gravity modulation parameter  $\epsilon \alpha$  leads to decrease in skin-friction  $\tau$ . The phase of skin-friction also increases with increase in value of gravity modulation parameter  $\epsilon \alpha$ .

4. Gravity modulation parameter has significant impact on Nusselt number. Even a small increase in gravity modulation parameter  $\epsilon \alpha$  leads to significant increase in the value of Nusselt number. Phase difference of Nusselt number decreases with increasing suction parameter, while increase in rarefaction parameter shows decrease in phase difference near the plate, but it shows reverse effect away from plate. With increasing  $Gr$  or  $Gc$ , for the same value of rarefaction parameter, Phase difference of Nusselt number decreases and the same effect is observed for increasing value of gravity modulation parameter  $\epsilon \alpha$ .

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## REFERENCES

- [1] K. Jules, K. Hrovat, E Kelly, K. McPherson and T. Reckart : International space station increment-2 microgravity environment summary report, *NASA 2002*, Report No. E-13146, NASA-TM-211335 (2002).
- [2] Y. Kamotami, A. Prasad and S., *Ostrich, AIAA J.*, **19**: 511-516 (1944).
- [3] M. Wadiah and B.Roux, *Journal of Fluid mechanics*, **193**: 391-415 (1988).
- [4] S.L. Biringen and J. Peltier, *Phys. Fluids A* (2): 754-760 (1990).
- [5] W.B.B. Duval and D. Jacquemin, *AIAA J.*, **28**: 1933-1941 (1990).
- [6] B. Ramaswamy, *Int. J of Numerical Methods for Heat & Fluid Flow*, **3**: 429-444 (1993).
- [7] R. Clever, IG. Schubert and F. H. Busses, *Physics of Fluids*, **5**: 2430-2437 (1993).
- [8] A. Farooq, and G.M. Homsy, *J Fluid Mech.*, **313**: 1-38 (1996).
- [9] M. Takako, H. Katsuya and T. Hirochika, *Trans. JSME*, **65**: 3054-3061 (1999).
- [10] C.F. Chen, and W.Y. Chen, *J Fluid Mech.*, **313**: 1-38 (1999).
- [11] D.A.S. ReesI. Pop, *Heat & Mass Transfer*, **37**: 403-08 (2001).
- [12] Y. Shu, B. Q. Li, and H.C. De Groh, *Numerical Hat Transfer Part A*, **39**: 245-65. (2001).
- [13] C.I. Christov and G.M. Homsy, *J. Fluid Mech.*, **430**: 335-60 (2001).
- [14] S. Sharidan, N. Amin and I. Pop, *Int. J. Heat and Mass Transfer*, **48**: 4526-40 (2005).
- [15] S. Sharidan, N. Amin and I. Pop, *Microgravity - Science and Technology*, **18**: 5-14 (2006).
- [16] V.K. Siddavaram and G.M. Homsy, *J. Fluid Mech.*, **562**: 445-75 (2006).
- [17] V.K. Siddavaram and G. M. Homsy, *J. Fluid Mech.*, **579**: 445-66 (2007).
- [18] M.P. Dyko and K. Vafai, *Int. J. Heat & Mass Transfer*, **50**: 348-60 (2007).
- [19] S. Sharidan, N. Amin and I. Pop, *Mechanics Reserch Communications*, **34**: 115-22 (2007).
- [20] P.K. Sharma, *Mathematicas Ensernaza Universitaria*, **13**: 51-62 (2005).